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2. Schulaufgabe	B12TS	17.02.11		

1.1 SF: $x = 2$ in Z: $-4 + 4k - 9 = 0 \Leftrightarrow k = 13/4$

$$f_{13/4}(x) = \frac{-(x^2 - 13/2\,x + 9)}{x-2} = \frac{-(x - 9/2)(x-2)}{x-2} = -x + 9/2$$

eine NST: $x_1 = 9/2$

Sonst: $D = 4k^2 - 36 = 4(k+3)(k-3) = 0$

- $k \in]-3;3[$: keine NST
- $k \in \{-3;3\}$: $x_{1/2} = \frac{-2k}{2\cdot(-1)} = k$: eine NST (do)
- $k \in \mathbb{R} \setminus [-3;3]$: (und $k \neq 13/4 = 3{,}25$)

$$x_{1/2} = -\frac{1}{2}\left(-2k \pm \sqrt{4(k^2-9)}\right) = k \pm \sqrt{k^2-9}$$; 2 einf. NST

1.2.1 $f_3(x) = f(x) = (-x^2+6x-9):(x-2) = -x+4+\frac{-1}{x-2}$

$-(-x^2+2x)$

$4x - 9$

$-(4x-8)$

-1

Schräge As: $y = -x+4$

Senkr. As: $x = 2$

1.2.2

$$f'(x) = \frac{-1x^2+4x-3}{1x^2-4x+4} \Rightarrow f'(x) \to -1 \text{ für } x \to \infty$$

Wert der Tangentensteigung nähert sich an Asymp. steig. an.

1.2.3

$$f''(x) = \frac{(x-2)^{\cancel{2}}(-2x+4) - (-x^2+4x-3)\cdot 2\cancel{(x-2)}}{(x-2)^3\cancel{(x-2)}} =$$

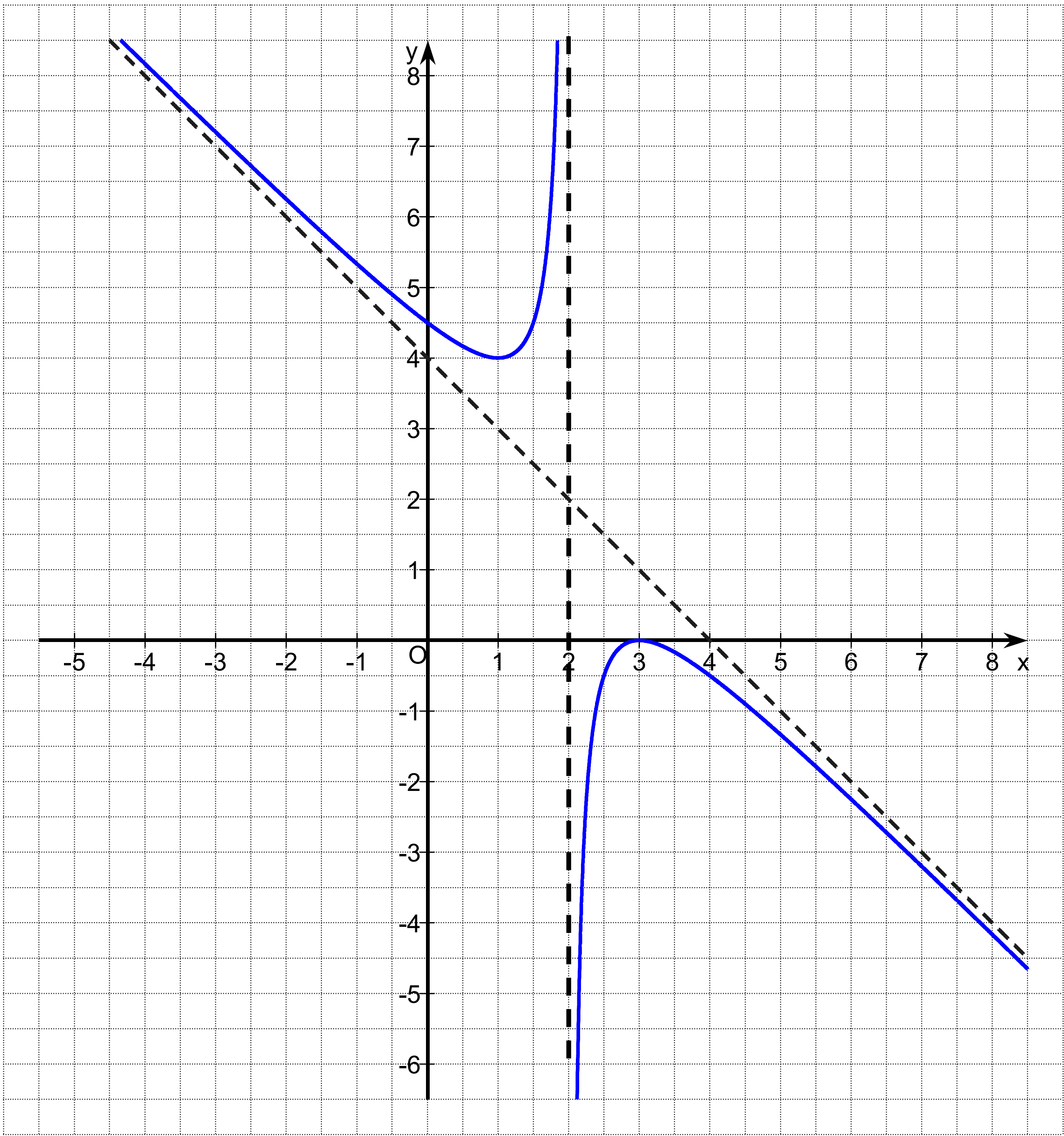
$$= \frac{-2x^2+4x+4x-8+2x^2-8x+6}{(x-2)^3} = \frac{-2}{(x-2)^3}$$

$f'(x) = 0 \Rightarrow -(x^2-4x+3) = -(x-3)(x-1) = 0$

$x_1 = 1$; $f''(1) = \frac{-2}{(1-2)^3} = 2 > 0 \Rightarrow$ TTP $(1/4)$; $f(1) = 4$

$x_2 = 3$; $f''(3) = \frac{-2}{(3-2)^3} = -2 < 0 \Rightarrow$ HOP $(3/0)$; $f(3) = 0$

1.2.4 Gf und Asymptoten



y
8
7
6
5
4
3
2
1
O
-5
-4
-3
-2
-1
1
2
3
4
5
6
7
8
x
-1
-2
-3
-4
-5
-6

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2.1

$$k\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}2a\\2\\2\end{pmatrix}\quad\begin{matrix}\Rightarrow a=-1\\ \Rightarrow k=-2\\ \Rightarrow k=-2\end{matrix}$$

Für ($k=-2$ und) $a=-1$ sind RV l.a $\Rightarrow$ par. od. id.?

$$\begin{aligned}-2&=0+\lambda &&\Rightarrow \lambda=-2\\ 0&=10-\lambda &&\Rightarrow \lambda=10 \;↯\\ 0&=-2-\lambda\end{aligned}$$

Für $a=-1$ sind die beiden Geraden g und h_a echt parall.

$$\left.\begin{aligned}0+\lambda&=-2+2\mu a\\ 10-\lambda&=0+2\mu\\ -2-\lambda&=0+2\mu\end{aligned}\right\}\ \text{III}-\text{II}:\ -12=0$$

Für $a\neq-1$ sind die Geraden windschief

2.2

$$\overrightarrow{RV_2}=\vec{a}_{h_a}-\vec{a}_g=\begin{pmatrix}0\\10\\-2\end{pmatrix}-\begin{pmatrix}-2\\0\\0\end{pmatrix}=\begin{pmatrix}2\\10\\-2\end{pmatrix}=2\cdot\begin{pmatrix}1\\5\\-1\end{pmatrix};\quad \overrightarrow{RV_2}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$$

$$\vec{n}^*=\begin{pmatrix}1\\-1\\-1\end{pmatrix}\times\begin{pmatrix}1\\5\\-1\end{pmatrix}=\begin{pmatrix}1+5\\-1+1\\5+1\end{pmatrix}=\begin{pmatrix}6\\0\\6\end{pmatrix}=6\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}\Rightarrow \vec{n}=\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$\vec{n}\circ(\vec{x}-a)=\begin{pmatrix}1\\0\\1\end{pmatrix}\circ\begin{pmatrix}x_1+2\\x_2-0\\x_3-0\end{pmatrix}=0\Rightarrow E:\ x_1+x_2+2=0$$

2.3

$$\overrightarrow{PX_\lambda}=\begin{pmatrix}0+\lambda-6\\10-\lambda-7\\-2-\lambda+2\end{pmatrix}=\begin{pmatrix}\lambda-6\\3-\lambda\\-\lambda\end{pmatrix};\quad \overrightarrow{PX_\lambda}\circ\vec{u}_g=\begin{pmatrix}\lambda-6\\3-\lambda\\-\lambda\end{pmatrix}\circ\begin{pmatrix}1\\-1\\-1\end{pmatrix}=0$$

$\Rightarrow \lambda-6-3+\lambda+\lambda=0 \Leftrightarrow 3\lambda=9 \Leftrightarrow \lambda=3$ in g:

$$\vec{l}=\overrightarrow{OL}=\begin{pmatrix}0\\10\\-2\end{pmatrix}+3\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}3\\7\\-5\end{pmatrix}\Rightarrow L(3\,|\,7\,|\,-5)$$

$$d=|\overrightarrow{PX_3}|=\left|\begin{pmatrix}3-6\\3-3\\-3\end{pmatrix}\right|=\left|\begin{pmatrix}-3\\0\\-3\end{pmatrix}\right|=\sqrt{9+0+9}=\sqrt{18}=3\sqrt{2}$$

$$=|\overrightarrow{PL}|$$

2.4

$$\overrightarrow{PL}=\begin{pmatrix}-3\\0\\-3\end{pmatrix}=-3\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}=3\cdot\vec{n}_E$$

$$\overrightarrow{PL}\perp E\Rightarrow d(P;E)=|\overrightarrow{PL}|$$

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2.5

$$\vec{u}_s = \overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix}6\\7\\-2\end{pmatrix} - \begin{pmatrix}0\\10\\-2\end{pmatrix} = \begin{pmatrix}6\\-3\\0\end{pmatrix}$$

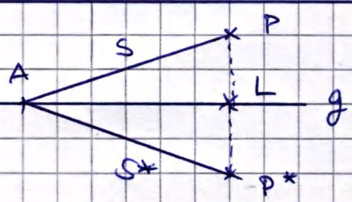
$$s: \vec{x} = \vec{a} + \beta \vec{u}_s = \begin{pmatrix}0\\10\\-2\end{pmatrix} + \beta \begin{pmatrix}6\\-3\\0\end{pmatrix} \; ; \; \beta \in [0;1]$$

$$\sin(\varphi) = \frac{\vec{u}_s \circ \vec{n}_E}{|\vec{u}_s| \cdot |\vec{n}_E|} = \frac{\begin{pmatrix}6\\-3\\0\end{pmatrix} \circ \begin{pmatrix}1\\0\\1\end{pmatrix}}{\sqrt{36+9}\,\sqrt{1+1}} = \frac{6}{\sqrt{45}\cdot\sqrt{2}} \left(= \frac{\sqrt{10}}{5}\right)$$

$$\varphi = \sin^{-1}\left(\frac{\sqrt{10}}{5}\right) \approx 39{,}23° \quad (\approx 0{,}6847 \text{ rad})$$

2.6

$$\vec{p}^{\,*} = \vec{\ell} + \overrightarrow{PL} = \begin{pmatrix}3\\7\\-5\end{pmatrix} + \begin{pmatrix}-3\\0\\-3\end{pmatrix} = \begin{pmatrix}0\\7\\-8\end{pmatrix}$$



$$\vec{u}_s^{\,*} = \overrightarrow{AP^*} = \begin{pmatrix}0\\7\\-8\end{pmatrix} - \begin{pmatrix}0\\10\\-2\end{pmatrix} = \begin{pmatrix}0\\-3\\-6\end{pmatrix}$$

$$s^*: \vec{x} = \vec{a} + \gamma \overrightarrow{AP^*} = \begin{pmatrix}0\\10\\-2\end{pmatrix} + \gamma \begin{pmatrix}0\\-3\\-6\end{pmatrix} \; ; \; \gamma \in [0;1]$$